# Implicatures in continuation-based dynamic semantics

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## Abstract

Advances have been made towards interpreting context-dependent meaning in a logical form, but treatments of implicatures remain incomplete. This paper captures implicature-related meaning in Lebedeva's (2012) extension of de Groote's continuation-based dynamic semantics (2006), exploiting the fact that context is incorporated as a parameter, meaning its structure may be altered while preserving the properties of the framework. The new context structure is a simple logic of common-sense reasoning using Poole's (1988) framework for classical logic that switches from *reasoning as deduction* to *reasoning as theory formation*. Focusing on *but* and supplementary content, a treatment of implicatures in a compositional framework – using only common tools from logic – is proposed. This is situated within the goal of formally accounting for presupposition, conversational implicature and conventional implicature in a single semantics.

## 1 Introduction

The *dynamic turn* in natural language semantics, attributed to Heim (1982) and Kamp (1981), relocated the meaning of a sentence from the logical form itself to its *context change potential*, interpreting new sentences in the context of those preceding. This enabled the interpretation of context-dependent meaning such as the referent of a pronoun, which had eluded the prevailing paradigm of Montague semantics (1970a; 1970b; 1973).

The dynamic semantics of de Groote (2006), as extended by Lebedeva (2012), goes further by incorporating *continuations* from programming language semantics (Strachey and Wadsworth, 1974) for a second notion of context as *the future of the discourse*. The result is a dynamic semantics in the style of Montague that firmly separates the context from the content of a sentence, uses only common mathematical tools – providing more insight than ad hoc definitions – and is entirely compositional – the meaning of a sentence is determined by the meanings of its constituents and its syntactic structure, allowing for the automatic interpretation of complex expressions. Furthermore, since both kinds of context are abstracted over the meaning of the sentence, the structure of the context is flexible – for example, a list of names (de Groote, 2006) or a conjunction of propositions (Lebedeva, 2012).

This paper exploits the flexibility of context by considering not just interaction with the context, but interaction *within* the context, to locate implicatures. Implicatures are situated in a group of meaning classes characterized by existing outside the plain semantic content of an utterance. Also in this group is presupposition – meaning assumed by an utterance for it to be meaningful – as in 'John quit smoking', which relies on John having smoked to make sense. If this presupposed information is not in the discourse context, it is accommodated alongside the plain content of the sentence. Implicature refers to meaning outside of what is explicitly said, logically entailed or presupposed by an utterance. It is traced back to Frege (1879) and was brought to prominence by Grice's (1975) treatment that introduced a provisional division – with prevailing terminology – between *conversational implicature*, governed by principles of cooperative conversation such as utterances being relevant to what has come before, and *conventional implicature*, instead associated with particular words – *but*, for example, is said to implicate a contrast between two clauses, while not explicitly stating this contrast.

If these meaning classes and their distinctions seem murky, it is because they are. Potts' (2015) survey of these phenomena contends that their definitions are "still hotly contested" and suggests refocusing towards developing "rich theories of properties... the way those properties interact, and the effects of those interactions on language and cognition." Lebedeva's extension of de Groote's framework goes some way towards this by accounting for presuppositions of referring expression and proposes a mechanism for handling conversational implicatures. Treatment within the same framework allows a preliminary formal distinction between presuppositions and certain kinds of conversational implicatures to be made.

This paper goes further by distilling Lebedeva's approach of *conversational implicatures by proof-theoretic abduction* to *implicatures by reasoning in the context*. By elaborating the context structure to a logical theory using Poole's (1988; 1989; 1990) classical logic framework for reasoning as theory formation, meaning associated with *conventional* implicatures is captured while preserving the features of compositionality and the use of common mathematical tools. In this paper, it is used to formalize an intuition about supplementary content, revealing proximity to conversational implicature, and provide a treatment of *but*.

Section 2 proceeds by detailing the problems of capturing implicatures. Section 3 provides the formal background: the continuation-based dynamic semantics in use, the approach to conversational implicatures by proof-theoretic abduction from which this work stems, and Poole's framework for reasoning as theory formation. Section 4 adapts this framework for natural language interpretation and uses the new context structure to solve the problems from Section 2.

## 2 Implicatures

This section gives a pre-formal presentation of implicatures and their challenges to formal semantics. Although divided into conversational and conventional varieties, the formalization will approach them in the spirit of Potts' aforementioned call to move from labels to rich theories of properties. Recall also that the solution we seek to these problems is one that is compositional and uses only common tools from logic, distinguishing it from other approaches.

### 2.1 Conversational implicature

Consider how A may interpret B's statement in the following discourse from Grice (1975):

(1) A: Smith doesn't seem to have a girlfriend these days.
B: He has been paying a lot of visits to New York lately.

Assuming that B is a cooperative speaker, providing content relevant to A's statement, B's response contains meaning outside of Smith's visits to New York. Suppose A believes having a girlfriend in a different city is a reason for frequently visiting that city, then A takes B to mean Smith has a girlfriend in New York. This is the conversationally implicated meaning Grice associates with (1) and is challenging to capture because it is not associated with a particular lexical item.

### 2.2 Conventional implicatures

Formalizing conventional implicatures is complicated by the fact that the term is used to refer to a diverse body of lexical items, has at least two very distinct characterizations, and is the subject of prominent claims of non-existence (Bach, 1999). The Gricean conventional implicatures (Grice, 1975) have been expanded to include adverbs *already, only, also, yet*; connectives *but, nevertheless, so, therefore*; implicative verbs *bother, manage, continue, fail*; and subordinating conjunctions *although, despite, even though*. We focus on *but* as a canonical example.

***But***

*But* is often thought of as contrasting two clauses, as in the following example from Bach (1999):

(2) Shaq is huge but he is agile.

Classical treatments of *but* follow a standard template for conventional implicatures observed in (Potts, 2015) of associating independent dimensions of meaning with a word. In the case of *but*, this is the pair $(p \wedge q, R(p, q))$, where $R$ represents a relation of contrast between $p$ and $q$.

The contrast need not be between the two clauses joined by *but*, however. The contrast in (3) is reasons for and against inviting Robinson:

(3) A: Robinson always draws large audiences.
B: He always draws large audiences, but he is in America for the year.

Returning to (2), Bach (1999) considers "the most natural way of taking *but* especially out of context" is "as indicating that being huge tends to preclude being agile". However, it is not clear whether "out of context" means by the conventional meaning of words alone, or additionally implies some knowledge of the world – that people exist in restricted spaces surrounded by objects that make swift movement easier for smaller people, or that it is biologically the case that great size generally precludes agility. To clarify, consider the following variations:

(4) Shaq is huge but he is rich.

(5) Shaq is huge but he is small.

Utterance (2) is comparable to (4) and different to (5) as only the latter contains a *conventional* contrast – based on the meaning of words alone – but is infelicitous for this very reason. Utterance (4) appears infelicitous "out of context", unlike (2), but not in a highly specific context: consider a conversation between Shaq's friends about who to invite on an expensive caving holiday. Speaker B suggests inviting Shaq, to which it is replied:

(6) A: Shaq is huge! He's too big to go caving.
B: Shaq is huge but he is rich.

The challenge is to account for these context-dependent conditions on felicitousness.

**Supplements**

The second characterization of conventional implicatures is Potts' (2005) reformulation, motivated by a dearth of formal treatments and based on Grice's remarks but divorced from the notion of implicature – enforced by called them 'CIs'. The formulation of CI as speaker-oriented commitments that are part of the conventional meaning of words and logically independent from at-issue content, is evidenced not by the classical examples above but expressives, such as 'damn' and supplemental expressions, underlined in the following example:

(7) Ed's claim, which is based on extensive research, is highly controversial.

While Potts' multidimensional logic for handling CIs spurred interest in formalizing this meaning class, it largely did not extend to Gricean conventional implicatures, and relationships to conversational implicatures remain unexplored, as in Potts' interpretation of (7):

> With the CI content expressed by the supplementary relative, I provide a clue as to how the information should be received. This example is felicitous in a situation in which, for example, I want to convey to my audience that the controversy should not necessarily scare us away from Ed's proposal – after all, it is extensively researched. (Potts, 2005)

The problem here is in explaining the proximity of Potts' description of CIs to Grice's notion of implicature as meaning outside of what is explicitly said, formalizing Potts' intuition that CIs provide "a clue as to how the information should be received".

## 3 Formal background

We proceed by introducing the natural language semantics to be used for these problems, the proposal for capturing conversational implicatures in this semantics by proof-theoretic abduction, and the framework for common-sense reasoning that will be used to generalize this approach.

### 3.1 Continuation-based semantics $GL\chi$

The continuation-based dynamic semantics $GL\chi$ (Lebedeva, 2012) is a version of de Groote's Montagovian account of dynamics (2006), enhanced by a systematic translation from static to dynamic interpretations and an exception raising and handling mechanism for capturing presuppositions. The interpretation of a sentence is a logical form in a $\lambda$-calculus, built compositionally from individual lexical items, such as the following:

$$\overline{[\![\textit{loves}]\!]} = \lambda \mathbf{YX}.\mathbf{X}(\lambda \mathbf{x}.\mathbf{Y}(\lambda \mathbf{y}.\overline{\mathbf{love}}\,\mathbf{x}\,\mathbf{y})) \quad (8)$$

$$\widetilde{[\![\textit{John}]\!]} = \lambda \mathbf{P}.(\mathsf{sel}(\text{named "John"})) \quad (9)$$

$$\widetilde{[\![\textit{Mary}]\!]} = \lambda \mathbf{P}.(\mathsf{sel}(\text{named "Mary"})) \quad (10)$$

Term (8) is analoguous to the static interpretation of *loves* in Montague semantics, except that $\overline{\mathbf{love}}$ abbreviates a systematic dynamization of $\mathbf{love}$ :

$$\begin{aligned}\overline{\mathbf{love}} =& \lambda e\phi.\mathbf{love}\,(\mathbf{x}e)(\mathbf{y}e)\\ &\wedge \phi(\mathsf{upd}(\mathbf{love}\,(\mathbf{x}e)(\mathbf{y}e), e))\end{aligned}$$

This is dynamic in the sense that it is parameterized by two contexts: $e$ is the *left context*, made of background knowledge and preceding sentences, and $\phi$ is the *right context*, made of the discourse to come. The right context is formally a *continuation* (Strachey and Wadsworth, 1974), invented for compositionality problems in the semantics of programming languages. Function $\mathsf{upd}$ adds new content to the context, while $\mathsf{sel}$, in terms (9) and (10), selects a referent from the context satisfying a certain property – such as being named "John".

With these terms, the sentence *John loves Mary* can be interpreted compositionally by $\beta$-reduction of the following term:

$$(\overline{[\![\textit{loves}]\!]}\ \widetilde{[\![\textit{Mary}]\!]})\widetilde{[\![\textit{John}]\!]} \to_\beta \lambda e\phi.$$
$$\textbf{love}\,(\mathsf{sel}(\text{named "John"})e)(\mathsf{sel}(\text{named "Mary"})e)$$
$$\wedge\ \phi(\mathsf{upd}(\textbf{love}\,(\mathsf{sel}(\text{named "John"})e)$$
$$(\mathsf{sel}(\text{named "Mary"})e), e)) \qquad (11)$$

Dynamic semantics is concerned with discourse, rather than individual sentences. Suppose we have a context containing Mary and John, formally:

$$\mathsf{c} = \exists j.\text{named "John"}j \wedge \exists m.\text{named "Mary"}m$$

Then interpretation in context $\mathsf{c}$ is found by applying the sentence-level interpretation (11) to $\mathsf{c}$, $\beta$-reducing and evaluating the oracle functions to find the referents of Mary and John:

$$\lambda\phi.\textbf{love}\,j\ m \wedge \phi(\mathsf{upd}(\textbf{love}\,j\ m, \mathsf{c})) \qquad (12)$$

If appropriate referents cannot be found, an exception is raising and handled by introducing new individuals to the context (see (Lebedeva, 2012) for further details).

Suppose the discourse continues with the sentence *He smiles at her*. Then it has the following interpretation, found by applying (12) to the sentence-level interpretation of *He smiles at her*:

$$\lambda\phi.\textbf{love}\,j\ m \wedge \textbf{smiles-at}\,j\ m$$
$$\wedge\ \phi(\mathsf{upd}(\textbf{smiles-at}\,j\ m, \mathsf{upd}(\textbf{love}\,j\ m, \mathsf{c})))$$

Since we will be using a context structure to capture implicatures, we are only interested in the last subterm of this expression – the incremental context update. In $GL\chi$, the context is treated as a conjunction of terms, so $\mathsf{upd}(\mathbf{t}, \mathsf{c})$ simply adds term $\mathbf{t}$ to context $\mathsf{c}$ by conjunction, as in:

$$\mathsf{upd}(\textbf{love}\,j\ m, \mathsf{c}) = \mathsf{c} \wedge \textbf{love}\,j\ m$$

Since context is defined as a parameter, its structure – and the definition of context update – may be changed while otherwise preserving the properties of the framework, including compositionality.

### 3.2 Conversational implicatures by proof-theoretic abduction

Our starting point for treating implicatures in framework $GL\chi$ is Lebedeva's (2012) proposal for conversational implicatures by proof-theoretic abduction. Abductive reasoning is adopting a statement because it provides an explanation for another statement known to be true: where deduction is the conclusion of $q$ from $p$ and $p \Rightarrow q$, abduction is the conclusion of $p$ from $q$ and $p \Rightarrow q$. Such reasoning is *defeasible*, in the sense of being open to revision.

Although logically invalid, abduction is prolific in human reasoning and Hobbs et al. (1993) argue that it is inherent in interpreting discourse, based on the hypothesis that "it is commonplace that people understand discourse so well because they know so much" (Hobbs et al., 1993). To interpret B's remark in (1) requires not just knowledge of the meaning of words but knowledge of the world – specifically that people spend time with their partners and seeing someone who lives elsewhere requires visiting them. This knowledge means reasoning occurs when new information is encountered, motivating the use of proofs to capture natural language meaning.

This is incorporated into $GL\chi$ via the definition of a handler for an exception raised when a proposition cannot be proved from the context of background knowledge and preceding sentences. This implements – in a compositional framework using only familiar logical tools – the idea from Hobbs (2004) of computing implicatures by attempting to prove the logical form of a sentence, taking as axioms formulae corresponding to the current knowledge base. If no proof is found, the facts necessary to complete the proof are added to the knowledge base via abduction. These abduced facts correspond to the implicatures of the sentence.

We develop this proposal in two ways. Firstly, the approach – left generic to demonstrate a concept – inherits the computational problems of both proof search and abduction, such as monotonicity. An implementation requires choosing a logic for abduction while preserving the original principles of the framework, namely the use of standard logical tools. To this end, we consider abduction *outside* of a proof-theoretic approach, observing that this is not intrinsic to the proposal and has the disadvantage of automatically excluding other ways of implementing abduction, such as a forward-reasoning system.

The second development is incorporating reasoning more broadly. Once one notion of reasoning has been introduced to the context, it becomes clear that interpretation can depend on deductive inference from content in the context, as well as *induction* – another form of defeasible reasoning. Inductive reasoning takes several cases of $p$ and $q$ occurring together to conclude $p \Rightarrow q$, and can be cast as *default reasoning* – as in '$q$ usually follows from $p$'. It is then necessary to account for how defeasible and non-defeasible information interact.

### 3.3 The *Theorist* framework

Based on this, we want a logic of defeasible reasoning with good computational properties and using familiar mathematical tools. For this, we choose Poole's logical framework for default reasoning (Poole, 1988), further developed in (Poole, 1989, 1990) and including an implementation called *Theorist*. It is a semantics for classical logic that considers reasoning not as deduction but as *theory formation*. This is achieved by allowing hypothetical reasoning, and so handles nonmonotonic reasoning in classical logic.

Given a standard first-order language over a countable alphabet, *formula* refers to a well-formed formula over this language and an *instance of a formula* refers to a substitution of free variables in a formula by terms in the language. The following sets are provided: $F$ of closed formulae thought of as 'facts', $\Delta$ and $\Gamma$ of (possibly open) formulae constituting the hypotheses – defaults and conjectures respectively – and $O$ of closed formulae of observations about the world.

The semantics has three definitions at its core. A *scenario* of $(F, \Delta \cup \Gamma)$ is a set $D \cup G$, where $D$ and $G$ are ground instances of elements of $\Delta$ and $\Gamma$ respectively, such that $D \cup G \cup F$ is consistent. An *explanation* of a closed formula $t$ from $(F, \Delta \cup \Gamma)$ is a scenario of $(F, \Delta \cup \Gamma)$ that implies $t$. An *extension* of $(F, \Delta)$ is the logical consequences of a maximal (with respect to set inclusion) scenario of $(F, \Delta)$, that is, the closure under modus ponens $\overline{F \cup D}$ for some maximal set $D$ of ground instances of $\Delta$. With these definitions in hand, a *state* of the system is a tuple $\langle F, \Delta, \Gamma, O, \mathcal{E} \rangle$ where $\mathcal{E}$ is the set of explanations of the observations in $O$.

Note that there can be multiple extensions of a scenario, and so a formula $g$ is *predicted by* $(F, \Delta)$ if $g$ is in every extension of $(F, \Delta)$. See (Poole, 1989) for other possible definitions of prediction and a discussion of different ways of computing explanations; we follow (Poole, 1990) in taking our explanations to be least presumptive (not implying other explanations) and minimal (not containing other hypotheses).

To illustrate, consider the following example from (Poole, 1989) of medical diagnosis. Suppose the starting state is $\langle F, \Delta, \Gamma, \{\}, \{\} \rangle$, with:

$$\begin{aligned} F &= \{\textbf{broken}(\text{tibia}) \Rightarrow \textbf{broken}(\text{leg})\} \\ \Delta &= \{\textbf{broken}(\text{leg}) \Rightarrow \textbf{sore}(\text{leg})\} \\ \Gamma &= \{\textbf{broken}(\text{leg}), \textbf{broken}(\text{tibia})\} \end{aligned}$$

If $\textbf{sore}(\text{leg})$ is observed, the new state is $\langle F, \Delta, \Gamma, \{\textbf{sore}(\text{leg})\}, \{E_{\text{leg}}\} \rangle$, where:

$$E_{\text{leg}} = \{\textbf{broken}(\text{leg}), \textbf{broken}(\text{leg}) \Rightarrow \textbf{sore}(\text{leg})\}$$

Another possible explanation is:

$$\begin{aligned} E_{\text{tibia}} = \{&\textbf{broken}(\text{tibia}), \\ &\textbf{broken}(\text{leg}) \Rightarrow \textbf{sore}(\text{leg})\} \end{aligned}$$

This is a minimal explanation, but not least presumptive.

Alternatively, suppose that from the initial state $\textbf{broken}(\text{leg})$ is observed. Then the new state is $\langle F, \Delta, \Gamma, \{\textbf{broken}(\text{leg})\}, \{\} \rangle$, and $\textbf{sore}(\text{leg})$ is predicted because it is in every extension.

## 4 Implicatures by reasoning in the context

With the formal background in place, we proceed by adapting Theorist for reasoning in natural language interpretation and use it to solve the problems from Section 2.

### 4.1 Theorist for implicatures

Using Theorist for implicatures requires categorizing the information in a discourse context. Defaults and conjectures play the same role in our application, while observations, with their incremental update, correspond naturally to content.

More difficult is the question of what constitutes fact – information that we are not prepared to give up. The intuition in a model-theoretic interpretation of knowledge about the world, such as 'Canberra is in Australia', is that it is not necessarily true in every model. In the case of natural language, there is information that must be true in every model – lexical semantic information. Meaning we are not prepared to give up is the meaning of words and relationships between them, such as antonyms and 'green is a colour'. Thus we take this to correspond to the facts in Theorist. A new set $B$ is added, corresponding to background knowledge and containing any individuals given a priori, or via $GL\chi$'s exception handling mechanism. These entities comprise the domain of the context, assumed to be pairwise distinct.

We can now make the following definitions. A *state* of the discourse context is a tuple

$$\langle L, \Delta, \Gamma, O, B, \mathcal{E}, \mathcal{P} \rangle$$

with sets of closed formulae $L$ of lexical semantic information, $B$ of background information and $O$ of discourse content; sets of open formulae $\Delta$ of defaults and $\Gamma$ of conjectures; and sets of sets of closed formulae $\mathcal{E}$ of explanations and $\mathcal{P}$ of predictions. Sets $L, B$ and $\Delta$ are provided by the user, and $\Gamma$ may be given automatically as the set of antecedents of the implications in $\Delta$.

Given a context $\mathsf{c} = \langle L, \Delta, \Gamma, B, O, \mathcal{E}, \mathcal{P} \rangle$, the context update function $\mathsf{upd}$ in $GL\chi$ is defined:

$$\mathsf{upd}(\mathbf{t}, \mathsf{c}) = \langle L, \Delta, \Gamma, B', O', \mathcal{E}', \mathcal{P}' \rangle$$

- The new discourse content $\mathbf{t}$ is added to the set of observations:

$$O' \coloneqq O \cup \{\mathbf{t}\}$$

- The background information is updated with deductive inference from lexical semantic knowledge and the new content:

$$B' \coloneqq B \cup (\overline{L \cup O'}) \setminus (L \cup O')$$

- The explanation set contains the least presumptive and minimal explanations of $O'$ from $(L \cup B, \Delta \cup \Gamma)$, which takes the form of instances $D \cup G$ of $\Delta \cup \Gamma$.

- For each explanation $E_i$ there is a corresponding prediction set $P_i$ in $\mathcal{P}'$ defined by:

$$P_i = \overline{S_i} \setminus S_i$$

  where $S_i$ is the union of the maximal set of ground instances of $\Delta$ over the domain, the new discourse content and background, and explanation $E_i$:

$$S_i = max(\Delta) \cup B' \cup O' \cup E_i$$

Note that predictions are not made from the set of lexical semantic information since its consequences are not defeasible. Instead, it is placed in the background. Note also conjectures are used in explanation but not in prediction. We will make reference to the *hypotheses* $H(\mathsf{c})$ of a context theory $\mathsf{c}$ – the union of the explanations and predictions.

We return to the problems from Section 2. The computation of sentence-level interpretations are omitted but can be found compositionally in $GL\chi$.

### 4.2 Interaction of supplementary content

To answer the questions about the supplement in sentence (7) we want to represent the following information: *a claim can be unresearched, an unresearched claim is typically controversial, a controversial claim is typically rejected.* Let the initial context be given by $\mathsf{c}_0 = \langle L, \Delta, \Gamma, B, \{\}, \{\}, \{\} \rangle$, with $L, B, \Delta$ and $\Gamma$ as follows:

$$L = \{\}$$
$$B = \{\exists \lambda e.(\text{named "Ed"})e, \lambda f.\mathbf{claim}\, f \wedge \mathbf{poss}\, e\, f\}$$
$$\Delta = \{\neg \mathbf{researched}\, x \Rightarrow \mathbf{controversial}\, x,$$
$$\neg \mathbf{researched}\, x \Rightarrow \mathbf{reject}\, x\}$$
$$\Gamma = \{\neg \mathbf{researched}\, x\}$$

Consider the sentence with the supplementary content removed:

(13) Ed's claim is highly controversial.

The context update term of its interpretation in context $\mathsf{c}_0$ is:

$$\phi(\mathsf{upd}(\mathbf{controversial}\, f, \mathsf{c}_0))$$

Computating the $\mathsf{upd}$ function call to get $\mathsf{c}_1$:

$$\mathsf{c}_1 = \langle L, \Delta, \Gamma, B, \{\mathbf{controversial}\, f\}, \{E_1\}, \{P_1\} \rangle$$

where $E_1 = \{\neg \mathbf{researched}\, f, \neg \mathbf{researched}\, f \Rightarrow \mathbf{controversial}\, f\}$ and $P_1 = \{\mathbf{reject}\, f\}$. Interpreting (13) in this context predicts that Ed's claim should be rejected, and proposes it is controversial because it is not well researched.

Now consider inclusion of the supplement. Suppose *which* is given the same interpretation as the plain discourse connective *and*, differentiated only by its syntax.[1] Then the context update term of its interpretation in context $\mathsf{c}_0$ is:

$$\phi(\mathsf{upd}(\mathbf{controversial}\, f, \mathsf{upd}(\mathbf{researched}\, f, \mathsf{c}_0)))$$

[1] This interpretation is not sufficient to capture the projection behaviour of supplements, however this is beyond the scope of this paper.

Let $\mathsf{c}_1 = \mathsf{upd}(\textbf{researched}\, f, \mathsf{c}_0)$. Then:

$$\mathsf{c}_1 = \langle L, \Delta, \Gamma, B, \{\textbf{researched}\, f\}, \{\emptyset\}, \{\emptyset\}\rangle$$

and there is no explanation or prediction in the theory of context. Computing the second context update:

$$\begin{aligned}\mathsf{c}_2 &= \mathsf{upd}(\textbf{controversial}\, f, \mathsf{c}_1)\\ &= \langle L, \Delta, \Gamma, B, \{\textbf{researched}\, f,\\ &\quad \textbf{controversial}\, f\}, \{\emptyset\}, \{\emptyset\}\rangle\end{aligned}$$

Again, there is no explanation or prediction.

Potts' meaning – not to dismiss Ed's claim on the basis of being controversial – can be located in the difference between the context with and without the supplementary content. To do this, we expand the notion of context change potential to allow comparison of theories of context, formalizing the meaning of Potts' "clue". Significantly, this meaning is not associated with *which*, and so need not be encoded in a lexical item.[2] This is consistent with Potts' treatment and diagnosis of CI, however, formalizes the interaction of supplementary content with main content in a way that looks like conversational implicature. Thus the formalism proves valuable in identifying different flavours of implicature at play.

### 4.3 *But*

Again, the connective in question is assigned the same interpretation as the plain discourse connective *and*, to demonstrate how the meaning associated with *but* emerges through reasoning in the context.

Interpreting utterance (3) shows how this approach can identify a contrast existing outside of the clauses connected by *but*. We want to represent the following context: *being popular is a reason for inviting someone, not being in Oxford is a reason for not being invited* and *if someone is in Oxford then they are not in America*. Let $\mathsf{c}_0 = \langle L, \Delta, \Gamma, B, \{\}, \{\}, \{\}\rangle$, with sets given as follows:

$$\begin{aligned}L =&\{\}\\ B =&\{\exists\lambda r.(\text{named “Robinson”})r, \textbf{male}\, r,\\ &\textbf{human}\, r\}\\ \Delta =&\{\textbf{popular}\, x \Rightarrow \textbf{invite}\, x,\\ &\textbf{invite}\, x \Rightarrow \textbf{in-oxford}\, x,\\ &\textbf{in-oxford}\, x \Rightarrow \neg\textbf{in-america}\, x\}\\ \Gamma =&\{\textbf{popular}\, x, \textbf{in-oxford}\, x\}\end{aligned}$$

The interpretation of (3) in context $\mathsf{c}_0$ includes the following subterm for updating the context:

$$\phi(\mathsf{upd}(\textbf{in-america}\, r, \mathsf{upd}(\textbf{popular}\, r, \mathsf{c}_0)))$$

Beginning with the innermost context update:

$$\begin{aligned}\mathsf{c}_1 =&\mathsf{upd}(\textbf{popular}\, r, \mathsf{c}_0)\\ =&\langle L, \Delta, \Gamma, B, \{\textbf{popular}\, r\}, \{\emptyset\}, \{P_1\}\rangle\end{aligned}$$

There is no explanation, but it is predicted that Robinson should be invited, and that he is in Oxford and not America:

$$P_1 = \{\textbf{invite}\, r, \textbf{in-oxford}\, r, \neg\textbf{in-america}\, r\}$$

Performing the second context update:

$$\begin{aligned}\mathsf{c}_2 =&\mathsf{upd}(\textbf{in-america}\, r, \mathsf{c}_1)\\ =&\langle L, \Delta, \Gamma, B, \{\textbf{popular}\, r, \textbf{in-america}\, r\},\\ &\{\emptyset\}, \{\emptyset\}\rangle\end{aligned}$$

There is no explanation for the new content and no predictions, because **invite** $r$ is no longer consistent with the context. The meaning of *but* is located in the context change from $\mathsf{c}_1$ to $\mathsf{c}_2$. Rather than creating a contradiction in the context, a serious problem in a classical logic, Poole's framework prevents contradiction, preserving monotonicity in the context logic while capturing the occurence of inconsistencies. By modelling the theory of context this way, hard-coding a contradiction into the intepretation of *but*, as in the classical interpretation, becomes redundant – the inconsistency automatically arise in the context theories joined by *but*.

This model suggests viewing *but* as a pragmatic choice of connective to licence an inconsistency from one context to the next. This is not a new idea: it is compatible with a procedural account of meaning (Blakemore, 1987), in which beyond determining truth conditions, connectives guide the inferences made by the hearer of an utterance.

[2] Encoding in the interpretation of the lexical item could be useful for the problem of automatically generating context, however.

Based on these observations, we propose the following *pragmatic* definition of *but*, in the sense that it is defined on the level of discourse interpretation, as opposed to the semantic interpretation of a lexical item. Suppose *but* conjoins propositions $\mathbf{S}_a$ and $\mathbf{S}_b$, with the following context updates:

$$\begin{aligned} \mathsf{c}_n &= \mathsf{upd}(\mathbf{a}\,, \mathsf{c}_{n-1}) \\ \mathsf{c}_{n+1} &= \mathsf{upd}(\mathbf{b}\,, \mathsf{c}_n) \end{aligned}$$

Then there exists $p \in H(\mathsf{c}_n)$ and $q \in \mathsf{c}_{n+1}$ such that $p \wedge q \vdash \bot$, that is, there is a defeasible contradiction.

To test this proposal, consider (2) and variations (5) and (4). The context can be formalised as $\mathsf{c}_0 = \langle L, \Delta, \Gamma, B, \{\}, \{\}, \{\}\rangle$, with sets given as follows:

$$\begin{aligned} L =& \{\forall x.\mathbf{huge}\, x \Leftrightarrow \neg\mathbf{small}\, x\} \\ B =& \{\exists\lambda s.(\text{named "Shaq"})s, \mathbf{male}\, s, \mathbf{human}\, s\} \\ \Delta =& \{\mathbf{huge}\, x \Rightarrow \neg\mathbf{agile}\, x\} \\ \Gamma =& \{\mathbf{huge}\, x\} \end{aligned}$$

In the interpretation of (2) in context $\mathsf{c}_0$, the following subterm updates the context:

$$\phi(\mathsf{upd}(\mathbf{agile}\, s, \mathsf{upd}(\mathbf{huge}\, s, \mathsf{c}_0)))$$

Evaluating the innermost context update:

$$\begin{aligned} \mathsf{c}_1 =& \mathsf{upd}(\mathbf{huge}\, s, \mathsf{c}_0) \\ =& \langle L, \Delta, \Gamma, B_1, \{\mathbf{huge}\, s\}, \{\emptyset\}, \{P_1\}\rangle \end{aligned}$$

There is no explanation, but there is a prediction and the background is updated:

$$\begin{aligned} B_1 &= \{\neg\mathbf{small}\, s\} \\ P_1 &= \{\neg\mathbf{agile}\, s\} \end{aligned}$$

The new context theory predicts that Shaq is not agile. Performing the second context update:

$$\begin{aligned} \mathsf{c}_2 =& \mathsf{upd}(\mathbf{agile}\, s, \mathsf{c}_1) \\ =& \langle L, \Delta, \Gamma, B_1, \{\mathbf{huge}\, s, \mathbf{agile}\, s\}, \{\emptyset\}, \{\emptyset\}\rangle \end{aligned}$$

As in the previous example, there is a contradiction between subsequent contexts, with $\neg\mathbf{agile}\, s \in H(\mathsf{c}_1)$ and $\mathbf{agile}\, s \in \mathsf{c}_2$.

Accounting for the infelicitousness of (5), the update from $\mathsf{c}_0$ to $\mathsf{c}_1$ is the same, but the update $\mathsf{c}_2$ is as follows:

$$\begin{aligned} \mathsf{c}_2 =& \mathsf{upd}(\mathbf{small}\, s, \mathsf{c}_1) \\ =& \langle L, \Delta, \Gamma, B_1, \{\mathbf{huge}\, s, \mathbf{small}\, s\}, \{\emptyset\}, \{P_1\}\rangle \end{aligned}$$

Since lexical semantic consequence is not defeasible, and so is added to the background rather than predicted, $\neg\mathbf{small}\, s$ remains in the context from $\mathsf{c}_1$ to $\mathsf{c}_2$. Rather than having a contradiction between contexts, the contradiction is within the context.

For (4), the update from $\mathsf{c}_1$ to $\mathsf{c}_2$ is:

$$\begin{aligned} \mathsf{c}_2 =& \mathsf{upd}(\mathbf{rich}\, s, \mathsf{c}_1) \\ =& \langle L, \Delta, \Gamma, B_1, \{\mathbf{huge}\, s, \mathbf{rich}\, s\}, \{\emptyset\}, \{P_1\}\rangle \end{aligned}$$

There is no contradiction between $\mathsf{c}_2$ and $\mathsf{c}_1$, and so the condition under which *but* is the pragmatic choice of connective is not satisfied. However, the context for discourse (6) could be given as *inviting Shaq is a possibility, caving in a remote area is expensive, being rich is a reason for inviting someone on an expensive trip, being huge tends to make caving difficult, being unable to go caving is a reason for not inviting someone on a caving trip.* Then when *he is rich* is added to the context, there will be an inconsistency between subsequent contexts, between a reason to invite Shaq and a reason against inviting Shaq. This illustrates the context-dependence of *but*, and how $GL\chi$ with reasoning in the context can account for it.

## 5 Conclusion

The thesis advanced in this paper is that implicature-related meaning – under various labels – can be located by incorporating reasoning into the discourse context. By elaborating the structure of context in de Groote and Lebedeva's continuation-based dynamic semantics with Poole's framework for reasoning as theory formation, implicature-related meaning can be interpreted compositionally and with the use of only standard logical tools.

It remains to continue testing this approach on other instances of implicature and to see how locating this meaning in the context can address the projection problem for implicatures, all towards the goal of formally comparing the properties of meaning labelled presupposition, conventional implicature, and conversational implicature, in a single framework.


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## References

Kent Bach. 1999. The myth of conventional implicature. *Linguistics and Philosophy*, 22(4):327–366.

Diane Blakemore. 1987. *Semantic Constraints on Relevance*. Blackwell.

Phillipe de Groote. 2006. Towards a Montagovian account of dynamics. *Semantics and Linguistics Theory*, 16.

Gottlob Frege. 1879. Begriffsschrift. In M. Beaney, editor, *The Frege Reader, 1997*. Oxford: Blackwell.

H. Paul Grice. 1975. Logic and conversation. In *Syntax and Semantics*, volume 3, pages 41–58.

Irene Heim. 1982. *The Semantics of Definite and Indefinite Noun Phrases*. Ph.D. thesis, University of Massachusetts at Amherst.

Jerry R. Hobbs. 2004. Abduction in natural language understanding. In Laurence R. Horn and Gregory Ward, editors, *The Handbook of Pragmatics*. Blackwell.

Jerry R. Hobbs, Mark E. Stickel, Douglas Appelt, and Paul Martin. 1993. Interpretation as abduction. *Artificial Intelligence*, 63:69–142.

Hans Kamp. 1981. A theory of truth and semantic representation. In Jeroen Groenendijk, Theo Janssen, and Martin Stokhof, editors, *Formal Methods in the Study of Language, Part 1*, volume 135, pages 277–322. Mathematical Centre Tracts, Amsterdam. Reprinted in Jeroen Groenendijk, Theo Janssen and Martin Stokhof (eds), 1984, Truth, Interpretation, and Information; Selected Papers from the Third Amsterdam Colloquium, Foris, Dordrecht, pp. 1–41.

Ekaterina Lebedeva. 2012. *Expressing discourse dynamics through continuations*. Ph.D. thesis, INRIA, Université de Lorraine.

Richard Montague. 1970a. English as a formal language. In B. Visentini et. al., editor, *Linguaggi nella e nella Tecnica*, pages 189–224. Edizioni di Comunità, Milan.

Richard Montague. 1970b. Universal grammar. *Theoria*, 36:373–398.

Richard Montague. 1973. The proper treatment of quantification in ordinary English. In J. Hintikka, J. Moravcsik, and P. Suppes, editors, *Approaches to Natural Language: proceedings of the 1970 Stanford workshop on Grammar and Semantics*, pages 221–242. Reidel, Dordrecht.

David Poole. 1988. A logical framework for default reasoning. *Artificial Intelligence*, 36(1):27–47.

David Poole. 1989. Explanation and prediction: An architecture for default and abductive reasoning. *Computational Intelligence*, 5(2):97–110.

David Poole. 1990. A methodology for using a default and abductive reasoning system. *International Journal of Intelligent Systems*, 5(5):521–548.

Christopher Potts. 2005. *The Logic of Conventional Implicatures*. Oxford University Press.

Christopher Potts. 2015. Presupposition and implicature. In Shalom Lappin and Chris Fox, editors, *The Handbook of Contemporary Semantics Theory*. Oxford: Wiley-Blackwell.

Christopher Strachey and Christopher P. Wadsworth. 1974. Continuations: A mathematical semantics for handling full jumps. Technical report, Oxford University, Computing Laboratory.