Two semantical conditions for superlative quantifiers

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Abstract

We discuss semantics of superlative quantifiers *at most n* and *at least n*. We argue that the meaning of a quantifier is a pair specifying a verification and a falsification condition for sentences with this quantifier. We further propose that the verification condition of superlative quantifiers has a disjunctive form, which should be interpreted in an epistemic way, that is as a conjunctive list of possibilities. We also present results of a reasoning experiment in which we analyze the acceptance rate of different inferences with superlative and comparative quantifiers in German. We discuss the results in the light of our proposal.

There is an on going debate concerning the right semantical interpretation of so-called superlative quantifiers, such as *at most n* and *at least n*, where n represents a bare numeral, e.g *two*. (Look inter alia: (Geurts & Nouwen, 2007), (Koster-Moeller et al, 2008), (Geurts et al., 2010), (Cummins & Katsos, 2010), (Nouwen, 2010), (Cohen & Krifka, 2011)). Generalized Quantifier Theory defines superlative quantifiers as equivalent to respective comparative quantifiers: *fewer than n* and *more than n*, that is:

$$at\,most\,n\,(A,B) \iff fewer\,than\,n+1(A,B) \quad (1)$$

$$at\,least\,n\,(A,B) \iff more\,than\,n-1(A,B) \quad (2)$$

It has been observed that in natural language those equivalences 1 and 2 might not hold, or at least they might not be accepted by language user based on pragmatical grounds. There are numerous differences between comparative and superlative quantifiers involving their linguistic use Geurts & Nouwen (2007), acquisition (Musolino, 2004), (Geurts et al., 2010), their processing (Koster-Moeller et al, 2008),(Geurts et al., 2010), as well as – what is our main focus – the inference patterns in which they occur. It has been for instance shown that majority of responders usually reject inferences from *at most n* to *at most n+*[1], although they accept the equivalent inference with comparative quantifiers (Geurts et al., 2010), (Cummins & Katsos, 2010).

In this paper we provide an algorithmic interpretation of superlative quantifiers that tries to explain the observed reasoning data. Furthermore we report on the results of a reasoning experiment that support our theoretical proposal.

We propose that the meaning of a quantifier as a pair $\langle C_F, C_V \rangle$, where C_V is a *verification condition* (specifies how to verify sentences with this quantifier) and C_F is a *falsification condition* (specifies how to falsify sentences with this quantifier). Verification and falsification conditions are to be understood *algorithmically* (as partial algorithms), with the "else" part of the *conditional instruction* being empty - thus, they verify (or falsify) the formulas only if their conditional test is satisfied. From a perspective of classical logic, these conditions should be dual, namely if C is a C_V condition for sentence ϕ, then C is a C_F condition for sentence $\neg\phi$, and vice versa. We further, however, observe that in the case of superlative quantifiers, there is a split between these two conditions. We suggest, that this bifurcation is a result of a pragmatic focus on the expressed borderline n.

[1] Here and below $n+$ denotes any natural number greater than n, while $n-$ denotes any natural number smaller than n.

1 Two semantic conditions for "at most"

Krifka (1999) points out that semantic interpretation is usually a pair that specifies when the sentence is true and when it is false. However, as he observes, a sentence *at most n x:* $\phi(x)$ says only that *more than n x:* $\phi(x)$ is false, and leaves a truth condition underspecified. In other words, the meaning of *at most n* provides an algorithm for falsifying sentences with this quantifier, but not (immediately) for verifying them. Consequently, the primal semantical condition of *at most n x:* $\phi(x)$ could be understood as an algorithm: "falsify when the number of x that are ϕ exceeds n", and would constitute what we understand by the falsification condition.

Definition 1 *(falsification condition for at most)*

$C_F(at\,most\,x : \phi(x)) := If\ \exists^{>n}x(\phi(x)),\ then\ falsify$

But how can we know when it is true? From the point of view of an algorithm, it is the "else" part of the conditional that should define the truth-condition. However a negation of a falsification condition is in sense *informationally empty*: it does not describe any concrete situation in which the given sentence can be verified. As a result, in those contexts that require to directly verify a sentence, we refer to a verification condition, which is specified independently. As expressing a positive condition, *at most n* may be understood as a disjunction *n or fewer than n* ("disjunctive *at most*").

$$\neg\exists^{>n}x\phi(x) \iff \exists^{<n+1}x\phi(x) \iff \exists^{=n}x\phi(x)\vee\exists^{<n}x\phi(x) \quad (3)$$

In order to define the verification condition, we adopt, following Zimmermann (2000), the view that disjunctive sentences in natural language are likely to get so-called epistemic reading that is they are interpreted as *conjunctive lists of epistemic possibilities*. According to the proposed solution a disjunction P_1 *or...or* P_n is interpreted as an answer to a question: *Q: What might be the case?* and, thus, is paraphrased as a (closed) list L: P_1 (might be the case) [and]... P_n (might be the case) (and nothing else might be the case). This results in the following reading of a disjunctive sentence:

Definition 2 *(Zimmermann, 2000)* $P_1\vee...\vee P_n \iff \diamond P_1\wedge...\wedge\diamond P_n\ and\ (closure)\ \forall P[\diamond P \rightarrow [P \between P_1\vee...\vee P \between P_n]]$

If we assume that disjunctions in natural language are likely to be interpreted as conjunctions of epistemic possibilities, then we get the following verification condition for *at most*:

Definition 3 *(epistemic interpretation of the verification condition for at most)*

$C_V^E(at\,most\,n\,x : \phi(x)) := If\ (\diamond\exists^{=n}x\phi(x)\wedge\diamond\exists^{<n}x\phi(x)), then\ verify$

$[and\ (closure)\ If\ \diamond\exists^{>n}x\phi(x),\ then\ falsify]$

The important point is the optional character of the closure. This bases on our assumption that the falsification and verification conditions are in a sense independent and only as a pair constitute the full semantic interpretation. Since the falsification condition, as defined in 3, is sufficient to account for the right semantical criterion of when the sentence with *at most n* is false, the closure of the verification condition is redundant and might or might not be considered in the reasoning process. The optional character of closure turns out crucial in evaluating validity of inferences with *at most n*.

It is easy now to observe that from $\diamond\exists^{=n}\phi(x)\wedge\diamond\exists^{<n}x\phi(x)$ one cannot infer $\diamond\exists^{=n+1}x\phi(x)\wedge\diamond\exists^{<n+1}x\phi(x)$: the conjunct $\diamond\exists^{=n+1}x : \phi(x)$ cannot be proven based on the premise, though it can be excluded only if the closure of the premise is applied. On the other hand, the inference: *n or fewer than n* → *n-1 or fewer than n-1* (in the epistemic interpretation) is blocked only due to closure of the conclusion. That is: $\diamond\exists^{=n}$ implied by the premise is contradicted by the closure of the conclusion, i.e. $\neg\diamond\exists^{>n-1}$. However, without the closure the implication holds (if the epistemic reading of the verification condition is applied).

2 "At least" and bare numerals

As an upward monotone quantifier, *at least n* appears to provide a clear verification algorithm: "verify when n x (that are ϕ) are found". Such a semantical interpretation would not, however, account for the linguistical differences between *at least n* and *more than n-1*.

Let us start with defining a falsification condition for *at least n* as follows:

Definition 4 *(falsification condition for at least)*

$C_F(at\ least\ n : x\phi(x)) := If\ \exists^{<n}x\phi(x),\ then\ falsify$

Defining a verification condition for "at least n" we first take into account following pragmatic focus that is put on the borderline n, which leads us to the disjunctive form of this quantifier: *(exactly)* *n or more than n*. Finally, we apply Zimmerman's epistemic interpretation.

Definition 5 (epistemic interpretation of the verification condition for *at least n*)

$C_F(at\ least\ n : x\phi(x)) := If\ (\diamond\exists^{=n}x\phi(x) \wedge \diamond\exists^{>n}x\phi(x)),\ then\ verify$

$and (closure)\ If\ (\bigvee_{i=0}^{n-1} \diamond\exists^{=i}x\phi(x)),\ then\ falsify$

Let us now show how the interpretation of the bare numeral n interacts with the validity of inferences (*n or more than n*)→ (*n-1 or more than n-1*), given the epistemic interpretation of disjunction. A bare numeral n (e.g. "two") can be interpreted as denoting any set of *at least* n elements, or a set of *exactly* n elements. Suppose now that n is interpreted with a closure: *exactly* n. It is easy to observe that, in such a case, *possible that n and possible that more than n* does not imply *possible that n-1 or possible that more than n-1*. The premise which is interpreted as in Definition 5 does not imply $\diamond\exists^{=n-1} \wedge \diamond\exists^{>n-1}$ (with closure $\bigwedge_{i=0}^{n-2} \neg \diamond \exists^{=i} x\phi(x)$) While $\diamond\exists^{>n-1}$ follows from both $\diamond\exists^{>n}$ and $\diamond\exists^{=n}$, the problematic element is $\diamond\exists^{=n-1}$, which is directly contradicted by the closure of the premise. But suppose that n does not get the "exact" reading, but it is interpreted barely as *there are n*. Then from *possible that n* we can infer *possible that n-1*, since the latter does not exclude the possibility that there is a bigger set of elements.

3 Main findings

In our pilot experiment on reasoning conducted on German native speakers: nearly 100% of responders accepted inferences from *at most n* to *not more than n* and vice versa, as well as from *n or fewer than n* to *at most n* (and vice versa), which suggests that they do see those expressions as equivalent. (Similarly for mutual inferences between: *at least n* and *not fewer than n*, and *at least n* and *n or more than n*). The inferences: *at least n*→ *at least n-* were accepted in only ca. 75% of cases, which suggests some, at least pragmatic mechanism, suppressing this inference. We propose that this rejection bases on the "exact" reading of bare numerals. It is worth to note that of subjects accepted inferences that base on the "at least" reading of bare numerals in almost 60% of cases, which highly correlated with their acceptance of the inferences: *n or more than n* → *n- or more than n-* ($p = .026$) and with their acceptance of inferences: *at least n* → *at least n-* ($p = .019$).

Furthermore, while inferences from *at most n* to *at most n+* were accepted only by 14% of responders, inferences from *not more than n* to *not more than n+* were already accepted by almost 32%. Thus, it seems that paraphrasing *at most n* to the negative form: *not more than n* facilitates the inference.

The results for the inferences with disjunctive forms of superlative quantifiers (*n or fewer than n* and *n or more than n*) are especially interesting. While logically valid inferences (*n or more than n*) → (*n- or more than n-*) are accepted by 65% people, the invalid inferences: (*n or more than n*) → (*n+ or more than n+*) are rather rejected (only 18% accept). The opposite effect, however, we get for disjunctive form of *at most*. The logically valid inferences (*n or fewer than n*) → (*n+ or fewer than n+*) are rather rejected (only 16% accept), whilst invalid inferences (*n or fewer than n*) → (*n- or fewer than n-*) are accepted in 39% of cases. The surprising result that subjects accepted the invalid inferences with "disjunctive at most" more frequently than the valid ones can be explained by our proposal. As we have proposed above, closure in the verification condition is optional, since the falsification condition is sufficient to account for the right semantics. However, if context enforces applying one of the semantical conditions (verification or falsification), then the other one might be ignored. While, from the perspective of classical logic it should be enough to use only one of the conditions (since the other can be defined via the first one), in the case of superlative quantifiers the epistemic reading of the verification condition creates the bifurcation in the meaning. This results in different inferential patterns in which those quantifiers occur, depending on what the context primarily enforced: the verification or falsification condition.

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